

MaSuD topic suggestions
2022-2023

1. The machine to play the sticks game

The game of sticks is simple: two players play with 8 sticks, each in turn removing one or two sticks, the one who takes the last one wins.

We have a machine with 8 cups numbered from 1 to 8. We initialise the machine with two balls of each colour (yellow/red) in each cup except in cup 1 with only two yellow balls.

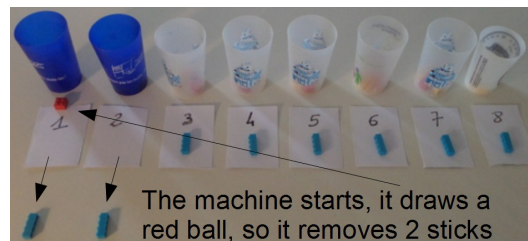
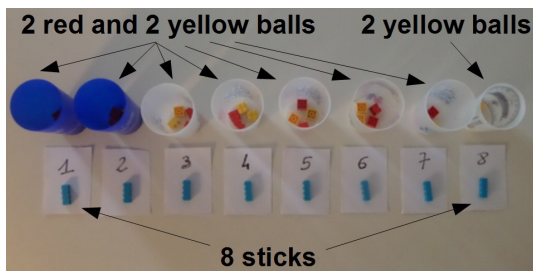
We suggest you play several games against a machine as follows:

- The machine starts.
- When it is the machine's turn to play, it draws a ball at random from the cup corresponding to the number of sticks left in the game. If the cup is empty, two yellow and two red balls are placed in the cup (except in cup n°1, where only two yellow balls are placed).
- If the drawn ball is yellow it removes one stick, otherwise two.
- The drawn balls are placed in front of the corresponding cup.

At the end of a game, if the machine has won, it is "rewarded" by putting the played balls back into the corresponding cups and for each ball that is put back, a ball of the same colour is added to the cup. If the machine loses, it is "punished" by removing the balls it has played.

On average, how many games does it take for the machine to always win?

How do you "program" a new machine if you change some of the parameters of the game: number of sticks, number of sticks you can remove (not necessarily consecutive numbers).



2. Weaving

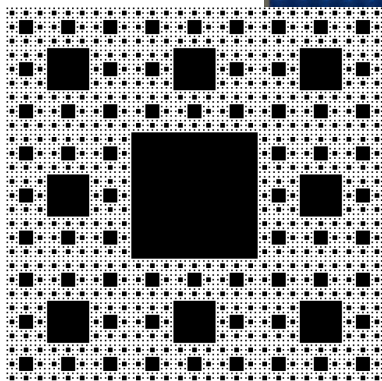
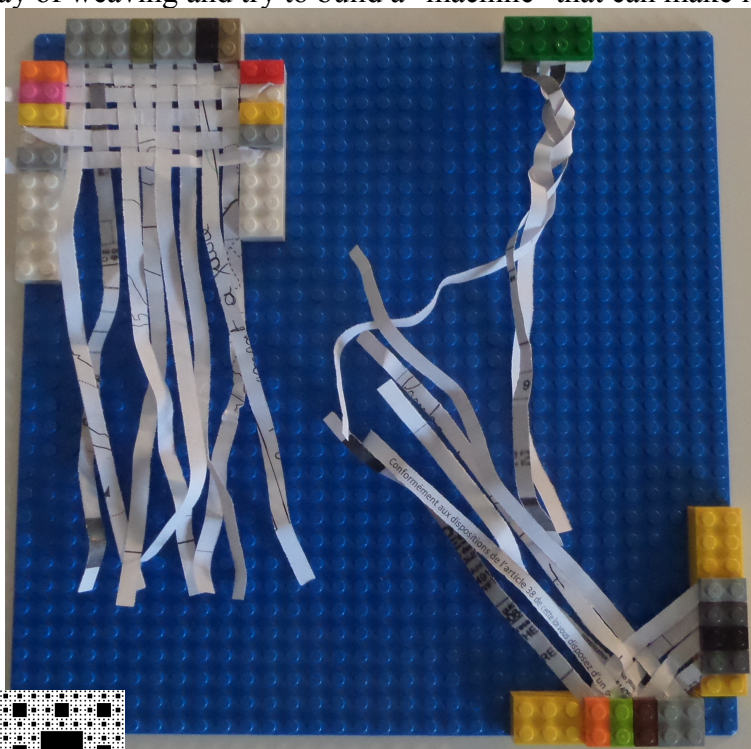
Strips of A4 paper are cut out (e.g. with a paper cutter) to obtain strips of identical width. With these strips of paper, we can weave (without glue).

Study the different types of braiding possible and make a 5cm strip as long as you want. Try to make strips, bags, hats or curtains with the used A4 sheets you have collected in your school.



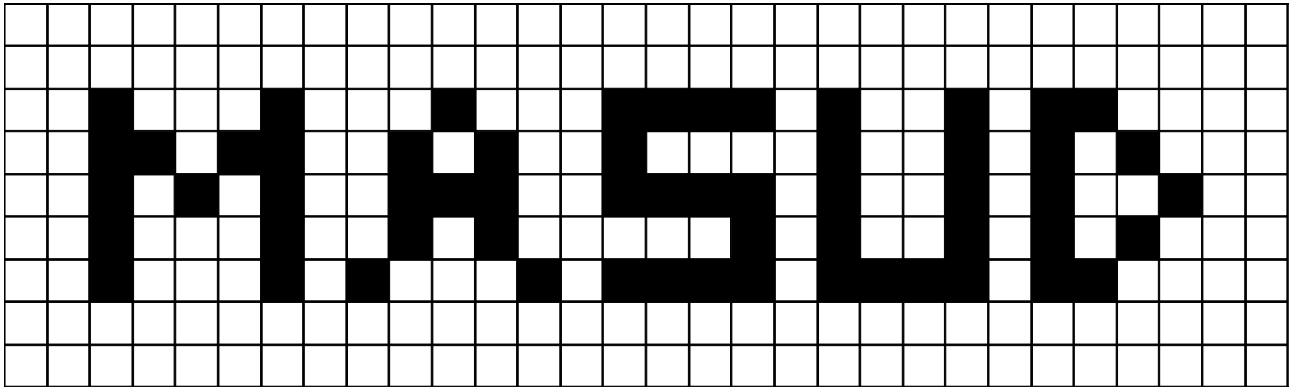
3. Weaving loom

Choose your own way of weaving and try to build a "machine" that can make it with paper strips.



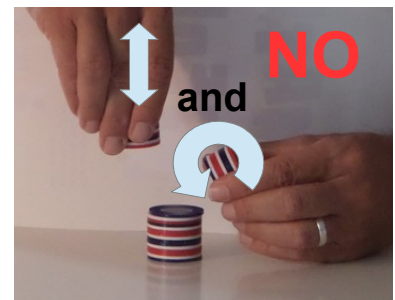
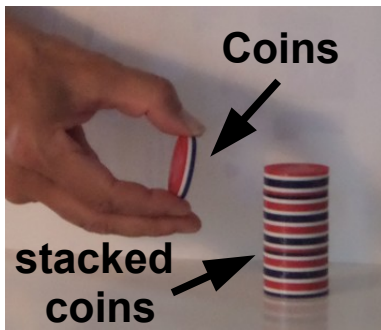
4. Make a pattern

We have identical strips of paper, one side white and the other black. We are going to weave to make a grid. How can we make a black and white image appear?



5. Turning over coins

We have a certain number of stacked coins. A stack of coins can be turned over partly, but only by starting from the top of the stack. How many times do you have to repeat this operation to get all the coins on the front side? How should we proceed in general?



6. Traffic jams

In order to simplify the problem of traffic jams, let's start with a simple case:

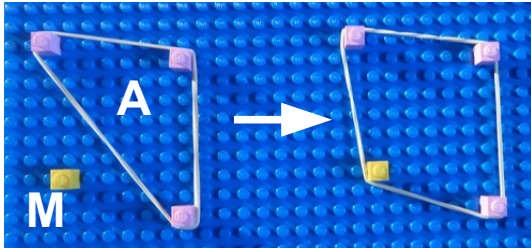
- a single line of cars
- the cars are all identical and move at the same speed
- two possible positions: stop or go
- a car moves forward one square when the space in front of it is empty
- a car stays in place when the space in front of it is occupied.



We place a number of cars on our road, to study the evolution of the traffic.

7. Inflated sets

We take a convex plane figure. The diameter of the figure is the greatest distance between two points of this figure. To this figure (A) we can "add" a point M outside the figure by considering the convex envelope formed by A and M. It is like placing a rubber band around A and M.

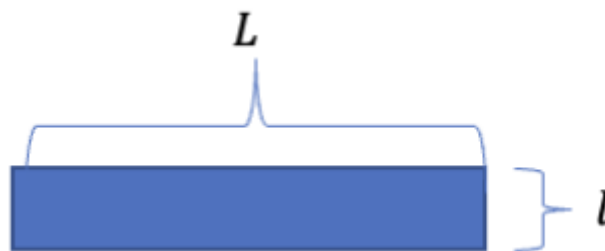


A figure is said to be "inflated" when the addition of any point in the plane (in the above sense) increases its diameter.

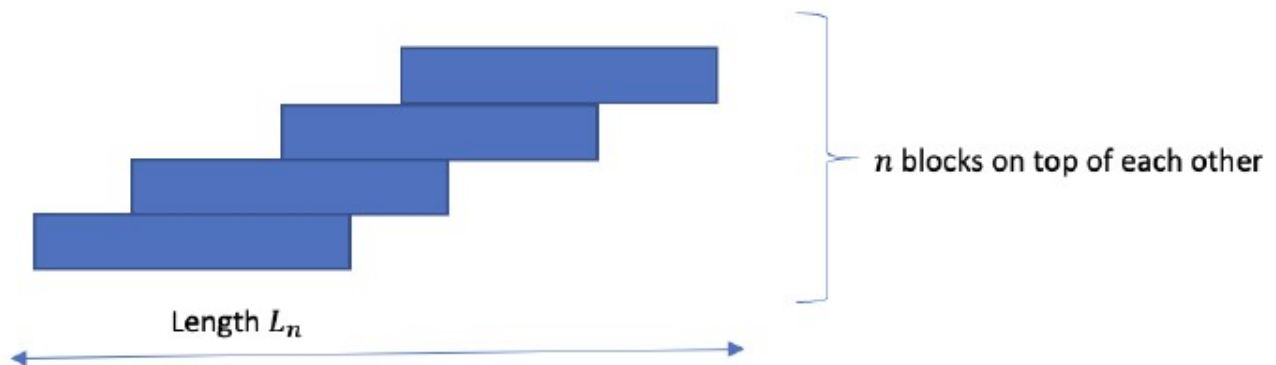
Try inflating a square and other shapes. What can be said about inflated figures of the same diameter?

8. The largest building

Suppose we have at our disposal building blocks of rectangular parallelepipedic shapes and of dimensions $L = 60$ cm and $l = 20$ cm and some depth $h = 10$ cm.



Can we place many of these blocks, on a planar floor, on top of each other without them collapsing such that the length L_n of this construction is 10 meters long horizontally?



If yes, how many pieces do we need for this? Is it possible to achieve a length L_n of 100 meters without the blocks collapsing? What is the minimal number of pieces n necessary for such a construction? What would be the vertical height of such constructions? Can you explore similar questions for building blocks of different shapes?

9. Evolution of parasites

In an isolated environment, we study the relation between a certain type of parasite and their host and how these evolve with time t (continuous or discrete). In our model, parasites deposit eggs on their hosts and when the eggs hatch, the host dies. Denote by H and P the number of hosts and parasites respectively (these can be modelled as a function of t). At each step (unit time), the number of eggs deposited depend on the probability that a parasite and a host meet. One can assume that this probability is proportional to the product $H \cdot P$ of the populations.

We are given fixed values b and d for the birth and death rate of hosts when no parasites are present. Moreover, we let dp be the death rate of the parasites.

Run simulations for given values of b , d and dp and try to determine what happens with the populations H and P in time.

10. Bacteria attacks

In a bacterial culture, some of the bacteria are producing a toxic substance that kills bacteria. The number $N(t)$ of bacteria organisms increases at each time unit in a way that is proportional to the existing population at any time and decreases at a rate proportional (per organism) to the concentration of the toxic substance. One can (but don't have to) assume that the change in population of bacteria is of the form

$$k \cdot N(t) \cdot (1 - a \cdot T_x(t))$$

where k , a are some fixed parameters and $T_x(t)$ is the concentration of toxic substance. Suppose the toxic substance is formed at a constant rate r per organism, hence the change in the concentration of toxic substance is of the form $r \cdot N(t)$

Perform experiments to analyse the evolution of the population $N(t)$ with time, for some given initial population N_0 , and fixed parameters k , a , r .

Computer implementations of these simulations would be very interesting.